THE UNIVERSITY OF HONG KONG

Department of Mathematics

Lecture Series

Model order reduction for dynamical systems

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Abstract

Model order reduction (MOR) are of growing importance in scientific computing as they provide a principled approach to approximate high-dimensional PDEs with low-dimensional models. Indeed, the dimensionality reduction provided by MOR help to reduce the computational complexity and time needed to solve large-scale, engineering systems [2, 1], enabling simulation based scientific studies not possible even a decade ago.

The MOR architecture is being exploited in many simulation based physics and engineering systems in order to render tractable many high-dimensional simulations. Fundamentally, the MOR algorithmic structure is designed to construct low-dimensional subspaces, typically computed with Singular Value Decomposition (SVD), where the evolution dynamics can be projected using a Galerkin method. Thus, instead of solving a high-dimensional system of differential equations (e.g. millions or billions of degrees of freedom), a low-rank model can be constructed in a principled way. Three steps are required for this low-rank approximation: (i) numerical solutions of the original high-dimensional system, (ii) dimensionality-reduction of this solution data typically produced with an SVD, and (iii) Galerkin projection of the dynamics on the low-rank subspace. The first two steps are often called the offline stage of the MOR architecture whereas the third step is known as the online stage. Offline stages are exceptionally expensive, but enable the (cheap) online stage to potentially run in real time.

A popular technique in MOR is the so-called Proper Orthogonal Decomposition (POD, see e.g. [3, 4]) which has been widely used in the scientific computing community. The primary challenge in producing the low-rank dynamical system is efficiently projecting the nonlinearity (inner products) to the POD basis, leading to numerous innovations in the MOR community for interpolating the projection. To give an idea of the power of this technique, for heat equation, it is possible to reduce the high dimensional system, say of dimension 103 into a lower one of dimension 5 with a high accuracy. It is then clear the computational efficiency of the method when reducing the dimension of the system. This feature is linked to the decay of the singular values.

In the course, we will discuss mathematical details of the POD method and its applications to dynamical and parametrized systems. Numerical examples will help to show the effectiveness of the method.

References

[1] P. Benner, S. Gugercin and K. Willcox. *A Survey of Projection-Based Model Reduction Methods for Parametric Dynamical Systems,* SIAM Rev. **57**, 2015, 483-531.

[2] A. Quarteroni and G. Rozza. *Reduced Order Methods for Modeling and Computational Reduction*, Springer, 2014.

[3] L. Sirovich. *Turbulence and the dynamics of coherent structures. Parts I-II,* Quarterly of Applied Mathematics, **XVL**, 1987, 561-590.

[4] S. Volkwein. Model Reduction using Proper Orthogonal Decomposition. Lecure Notes, University of Konstanz, 2013.

Lecture 1:	**January 18, 2019 (Friday), 4:00 – 5:00pm**
Lecture 2:	**January 21, 2019 (Monday), 4:00 – 5:00pm**
Lecture 3:	**January 25, 2019 (Friday), 4:00 – 5:00pm**

All lectures will be held in Room 210, Run Run Shaw Building, HKU

All are welcome